2015 (H) Ex. 8 V-Imp Exercise D.I.(H) Maths ①

paper-1st, Gr-A

W.U.

Summation of series

1(i) Find the sum of

$\cos\theta - \frac{1}{2}\cos 2\theta + \frac{1}{3}\cos 3\theta - \cdots$ (C+iS) method (G.S. single)

माना $C = \cos\theta - \frac{1}{2}\cos 2\theta + \frac{1}{3}\cos 3\theta - \cdots$

तथा $S = \sin\theta - \frac{1}{2}\sin 2\theta + \frac{1}{3}\sin 3\theta - \cdots$

$\therefore C+iS = (\cos\theta + i\sin\theta) - \frac{1}{2}(\cos 2\theta + i\sin 2\theta)$
$+ \frac{1}{3}(\cos 3\theta + i\sin 3\theta) - \cdots$

or $C+iS = e^{i\theta} - \frac{1}{2}e^{2i\theta} + \frac{1}{3}e^{3i\theta} - \cdots$

माना $x = e^{i\theta}$

$\therefore C+iS = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \cdots$

$= \log(1+x) = \log(1+e^{i\theta})$

$= \log(1+\cos\theta + i\sin\theta)$

$= \log\{2\cos^2\theta/2 + i\,2\sin\theta/2\cos\theta/2\}$

$= \log\{2\cos\theta/2(\cos\theta/2 + i\sin\theta/2)\}$

$= \log 2\cos\theta/2 \cdot e^{i\theta/2}$

$= \log(2\cos\theta/2) + \cancel{\log e^{\frac{i\theta}{2}}}\ \log e^{i\theta/2}$

$C+iS = \log(2\cos\theta/2) + i\theta/2\log e = \log 2\cos\theta/2 + i\theta/2 \cdot 1$

$C+iS = \log(2\cos\theta/2) + i\theta/2 \cdot 1$

क्यों $\log(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \cdots$

Equating real and imaginary parts, we have

$C = \log(2\cos\theta/2)$ and $S = \theta/2$

$\therefore C = \log(2\cos\theta/2)$ Ans

Summation of series

Formulae (S2)

Q.No 2. Find the sum of

$\sin\theta\cos\theta - \frac{1}{3}\sin^3\theta\cos3\theta + \frac{1}{5}\sin^5\theta\cos5\theta \cdots$ to ∞

Let $C = \sin\theta\cos\theta - \frac{1}{3}\sin^3\theta\cos3\theta + \frac{1}{5}\sin^5\theta\cos5\theta \cdots\cdots \infty$

and $S = \sin\theta\cdot\sin\theta - \frac{1}{3}\sin^3\theta\sin3\theta + \frac{1}{5}\sin^5\theta\sin5\theta \cdots \infty$

$C+iS = \sin\theta(\cos\theta + i\sin\theta) - \frac{1}{3}\sin^3\theta(\cos3\theta + i\sin3\theta) +$
$+ \frac{1}{5}\sin^5\theta(\cos5\theta + i\sin5\theta) \cdots\cdots$

$= \sin\theta\cdot e^{i\theta} - \frac{1}{3}\sin^3\theta\cdot e^{i3\theta} + \frac{1}{5}\sin^5\theta\, e^{5i\theta} \cdots\cdots$

$= x - \frac{1}{3}x^3 + \frac{1}{5}x^5 \cdots\cdots$ [माना $x = \sin\theta e^{i\theta}$

$= \tan^{-1}(x)$ .. By Gregory's series

$\therefore C+iS = \tan^{-1}(\sin\theta\, e^{i\theta})$ —— (1)

$\therefore C-iS = \tan^{-1}(\sin\theta\, e^{-i\theta})$ —— (2)

Adding (1) and (2) we get

$2C = \tan^{-1}(\sin\theta\, e^{i\theta}) + \tan^{-1}(\sin\theta\, e^{-i\theta})$

$= \tan^{-1}\dfrac{\sin\theta\cdot e^{i\theta} + \sin\theta\, e^{-i\theta}}{1 - \sin\theta\, e^{i\theta}\cdot\sin\theta\, e^{-i\theta}}$

or, $2C = \tan^{-1}\dfrac{\sin\theta(e^{i\theta} + e^{-i\theta})}{1 - \sin^2\theta \times e^0}$

$= \tan^{-1}\dfrac{\sin\theta(2\cos\theta)}{\cos^2\theta}$

$2C = \tan^{-1}\dfrac{2\sin\theta}{\cos\theta} = \tan^{-1}(2\tan\theta)$

$\therefore C = \frac{1}{2}\tan^{-1}(2\tan\theta)$ Ans

3 (3)

W.O

$$\cos\theta + \frac{a\cos 2\theta}{\lfloor 1} + \frac{a^2\cos 3\theta}{\lfloor 2} + \frac{a^3\cos 4\theta}{\lfloor 3} + \cdots \quad (a<1)$$

Let $c = \cos\theta + \frac{a\cos 2\theta}{\lfloor 1} + \frac{a^2\cos 3\theta}{\lfloor 2} + \frac{a^3\cos 4\theta}{\lfloor 3} + \cdots$

and $s = \sin\theta + \frac{a\sin 2\theta}{\lfloor 1} + \frac{a^2\sin 3\theta}{\lfloor 2} + \frac{a^3\sin 4\theta}{\lfloor 3} + \cdots$

$\therefore c + is = (\cos\theta + i\sin\theta) + \frac{a}{\lfloor 1}(\cos 2\theta + i\sin 2\theta) +$

$a^m \cdot a^n = a^{m+n}$

$\frac{a^2}{\lfloor 2}(\cos 3\theta + i\sin 3\theta) + \frac{a^3(\cos 4\theta + i\sin 4\theta)}{\lfloor 3} + \cdots$

$\therefore c + is = e^{i\theta} + \frac{a}{\lfloor 1} e^{2i\theta} + \frac{a^2}{\lfloor 2} e^{3i\theta} + \frac{a^3}{\lfloor 3} e^{i4\theta} + \cdots$

$= e^{i\theta}\left\{1 + \frac{ae^{i\theta}}{\lfloor 1} + \frac{a^2e^{2i\theta}}{\lfloor 2} + \frac{a^3e^{3i\theta}}{\lfloor 3} + \cdots\right\}$

Let $x = ae^{i\theta}$

$= e^{i\theta}\left\{1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \cdots\right\}$

$= e^{i\theta} \cdot e^{x} = e^{i\theta} \cdot e^{ae^{i\theta}}$

$= e^{i\theta} \cdot e^{a(\cos\theta + i\sin\theta)}$

$= e^{i\theta + a\cos\theta + ia\sin\theta}$

$= e^{a\cos\theta + i(\theta + a\sin\theta)}$

$= e^{a\cos\theta} \cdot e^{i(\theta + a\sin\theta)}$

$c + is = e^{a\cos\theta}\left[\cos(\theta + a\sin\theta) + i\sin(\theta + a\sin\theta)\right]$

Equating real parts both sides, we get

$c = e^{a\cos\theta}\cos(\theta + a\sin\theta)$ Ans

NB. $e^x = 1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \cdots$

$e^{i\theta} = \cos\theta + i\sin\theta$

$e^{-i\theta} = \cos\theta - i\sin\theta$

$\therefore e^{i\theta} + e^{-i\theta} = 2\cos\theta$